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Dispersion of Light and the Geometric Structure of the Universe

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Abstract We investigate the possible use of photon dispersion mechanisms in cosmology. In particular, we consider ordinary dispersion in a cold electron plasma, as well as recently proposed photon dispersion due to vacuum quantum fluctuations. We also consider dispersion due to a finite photon mass, for comparison. The dispersion time delay of radiation from extragalactic sources such as gamma-ray bursts (GRBs), their afterglows and flares in active galactic nuclei is estimated for the various mechanisms and the results compared. Finally it is shown how the delay can be used in a new cosmological test to differentiate between world models.

1. Introduction

Soon after the discovery of pulsars it was pointed out that if an extragalactic pulsar could be detected, dispersion time delay might be used to put a strong upper limit on the photon mass [11]. A few years later Ginzburg [12] suggested that the density of free electrons in the intergalactic medium (IGM) could be estimated by observing dispersion time delays in cosmic GRBs that had been reported for the first time that same year [18], provided the bursts were cosmological. Twenty years later Palmer [23] pointed out that if GRBs emitted a small fraction of their energy in the radio spectrum, measurements of delayed radio pulses might be used to determine the distances to the bursts. Lipunova et al. [19] then suggested that the dispersion of low frequency radio waves from a GRB might make the source observable as a delayed radio afterglow. A search for prompt GRB counterparts was attempted at 74 MHz with the FLIRT radio telescope but with ambiguous results [2].

A few years ago Amelino-Camelia et al. [1] pointed out that quantum gravity (QG) may cause modification of the dispersion relation for photons due to their interaction with vacuum quantum fluctuations, and that measurements of time delays in the radiation from GRBs and other explosive extragalactic events might be used to set bounds on the energy scale of quantum gravity, $E_{\rm QG}$. More recently Choubey & King [7] have applied similar ideas to a possible QG dispersion of neutrinos.

Measurements of time delays in the radiation from GRBs and other extragalactic sources have already been attempted and the results used to put limits on $E_{\rm QG}$ [9, 4, 17, 10] as well as various other fundamental quantities, such as the photon mass, m_γ, and the fractional variation in the speed of light with frequency, $\Delta c/c$ (see e.g. Schaefer [26] and references therein).

In this paper we shall not be concerned with questions of the type discussed above. Instead we will show that almost independently of the dispersion mechanism, the resulting time delay of radiation from cosmological sources can, at least in principle, be used as a new cosmological test to differentiate between world models. Similar ideas have previously been discussed in a less systematic way [28], but only in the case of dispersion due to quantum gravity effects. In §2 we set the stage with a short general discussion on dispersion. We then review in turn the ordinary dispersion in cold plasmas (§3), the dispersion due to a finite photon mass (§4) and finally the proposed dispersion due to quantum gravity (§5). The new cosmological test will be discussed in detail in §6 and our conclusions are given in §7.

2. General formalism

The group velocity, u, of a propagating electromagnetic pulse depends both on its frequency and the dispersion mechanism. For our purposes it is convenient to write u as

$$u = u(\nu, \nu_\star) = c\,[1 - f(\nu, \nu_\star)]^{\alpha} \ , \tag{1}$$

where c denotes the usual vacuum light speed, ν is the frequency of the photon and $\nu_\star$ is a characteristic frequency, which depends on the type of dispersion. The dimensionless function $f = f(\nu, \nu_\star)$ and the number α also depend on the dispersion mechanism.

If a photon travels a distance L from a given source to Earth and only one dispersion mechanism is operating, its propagation time is given by

$$t_\star = \int_0^L \frac{ds}{u} = \frac{1}{c}\int_0^L [1 - f(\nu, \nu_\star)]^{-\alpha} ds \ , \tag{2}$$

where we have assumed that the source is not cosmological. The effects of the expanding universe as well as various cosmological models will be discussed in §6. In most cases of interest $\mid f(\nu, \nu_\star) \mid \ll 1$ and hence we may write

$$t_\star \approx \frac{L}{c} + \frac{1}{c}\int_0^L \alpha f(\nu, \nu_\star) ds \ . \tag{3}$$

Two pulses with different frequencies, ν_1 and ν_2, leaving the source simultaneously, arrive at Earth at different times. When Eq. (3) applies, the time difference is simply given by

$$\Delta t_\star = t_{\star 1} - t_{\star 2} \approx \frac{1}{c}\int_0^L \alpha\,[f(\nu_1, \nu_\star) - f(\nu_2, \nu_\star)]\, ds \ . \tag{4}$$

Thus by measuring $\Delta t_\star$ for different sources with known L one can infer something about $\nu_\star$, α and f and thus obtain information about the dispersion mechanism. Conversely, if the dispersion mechanism is known as well as the distance to the source, one can obtain information about the microscopic processes that cause the dispersion (§3, 4, 5). Time delay measurements also provide limits on possible variations in the speed of light.

As already indicated, our main objective in this paper is to investigate dispersion time delay in the cosmological context. In particular we shall show how the delay can be used as a basis for a new cosmological test, *dispersion time delay versus redshift*. This will be discussed in detail in §6.

3. Ordinary dispersion

An electromagnetic signal with frequency ν propagates through a tenuous cold electron plasma with group velocity

$$u_{\rm p} = c\left(1 - \left(\frac{\nu_{\rm p}}{\nu}\right)^2\right)^{1/2} , \tag{5}$$

provided $\nu > \nu_{\rm p}$, where

$$\nu_{\rm p} = \left(\frac{n_{\rm e} e^2}{\pi m_{\rm e}}\right)^{1/2} = 8.98 \times 10^3 \ n_{\rm e}^{1/2} \ {\rm Hz} \tag{6}$$

is the plasma frequency, $n_{\rm e}$ denotes the density of free electrons in the plasma and e and $m_{\rm e}$ are the charge and the mass of the electron respectively (see e.g. chapter 7 in Jackson [16]). In Eq. (6) and below, cgs-units are used.

Comparing Eqs. (5) and (1) with $\nu_\star = \nu_{\rm p}$ we see that

$$\alpha f(\nu, \nu_{\rm p}) = \frac{1}{2}\left(\frac{\nu_{\rm p}}{\nu}\right)^2 \tag{7}$$

and for $\nu_2 > \nu_1 \gg \nu_\mathrm{p}$ we find from Eq. (4) that the travel time difference is given by

$$\Delta t_\mathrm{p} \approx \frac{1}{2c} \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) \int_0^L \nu_\mathrm{p}^2 \, ds \; . \tag{8}$$

Putting all the directly observable quantities on the left hand side we can rewrite Eq. (8) in the form

$$\Delta t_\mathrm{p} \, \nu_1^2 \left[1 - \left(\frac{\nu_1}{\nu_2} \right)^2 \right]^{-1} \approx \frac{1}{2c} \int_0^L \nu_\mathrm{p}^2 \, ds = \frac{e^2}{2\pi m_\mathrm{e} c} \int_0^L n_\mathrm{e} \, ds \; . \tag{9}$$

We now discuss this dispersion in more detail, first for the interstellar medium (ISM) and then for the intergalactic medium (IGM).

3.1. The interstellar medium

In addition to an overall dependence on distance and direction from the Galactic center, the free electron density in the interstellar medium (ISM) is known to be highly inhomogeneous [27, 6, 15]. As a result the interstellar electron column density is anisotropic as seen from an observer on Earth. However, for the purpose of making order of magnitude estimates we shall neglect these complications and set n_e equal to the mean value of the electron number density in our Galaxy [22]:

$$< n_\mathrm{e} >_\mathrm{ISM} \approx 0.03 \ \mathrm{cm}^{-3} \; . \tag{10}$$

From this it follows that the mean plasma frequency is given by

$$< \nu_\mathrm{p} >_\mathrm{ISM} \approx 1.6 \ \mathrm{kHz} \left(\frac{< n_\mathrm{e} >_\mathrm{ISM}}{0.03 \ \mathrm{cm}^{-3}} \right)^{1/2} \; . \tag{11}$$

The wavelength corresponding to $1.6 \ \mathrm{kHz}$ is 190 km. In more detailed work one must of course take into account the full dependence $n_\mathrm{e} = n_\mathrm{e}(\boldsymbol{r})$, where $\boldsymbol{r}$ is the position vector.

Inspection of Eq. (8) shows that the greatest time difference occurs if ν_1 is as low as possible and ν_2 $(> \nu_1)$ as large as possible. Reflection from the ionosphere sets a lower bound of $\nu_1 \approx$ 20–30 MHz for observations from the Earth's surface. Relevant Galactic distances are of order kpc and hence we write [22]

$$\left(\frac{\Delta t_\mathrm{ISM}}{\mathrm{min}} \right) \approx 2.3 \ \left(\frac{\nu_1}{30 \ \mathrm{MHz}} \right)^{-2} \left[1 - \left(\frac{\nu_1}{\nu_2} \right)^2 \right] \left(\frac{L}{\mathrm{kpc}} \right) \left(\frac{< n_\mathrm{e} >_\mathrm{ISM}}{0.03 \ \mathrm{cm}^{-3}} \right) \; . \tag{12}$$

Using this result for a signal from the Galactic edge together with the model of Taylor & Cordes [27] for $n_\mathrm{e} = n_\mathrm{e}(\boldsymbol{r})$, we find that a high frequency pulse should lead a low frequency radio pulse by minutes in directions perpendicular to the Galactic plane, whereas the time difference may be of the order of hours if the pulses travel along the Galactic disk.

3.2. The intergalactic medium

Although little is known about the distribution of free electrons in the IGM it is expected to be highly inhomogeneous (see e.g. chapter 23 in [24] as well as [21] and references therein). One can, however, estimate the mean electron density from the cosmological mass density as follows: Recent observations indicate that $\Omega_\mathrm{b0} h_0^2 \approx 0.02$ (see e.g. [5]). Here Ω_b0 is the current baryonic density parameter and $h_0 = H_0/100 \ \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$ is the dimensionless Hubble parameter. Assuming complete ionization, $x \sim 1$, at least out to a redshift of $z \sim 6$ (see e.g. [20] and references therein) we find that the mean density of free electrons as a function of redshift z is given by:

$$< n_\mathrm{e} >_\mathrm{IGM} (z) = x \frac{\Omega_\mathrm{b0} \rho_\mathrm{c}}{m_\mathrm{H}} (1+z)^3 \approx 2.2 \times 10^{-7} \ \mathrm{cm}^{-3} \left(\frac{x \Omega_\mathrm{b0} h_0^2}{0.02} \right) (1+z)^3 \; , \tag{13}$$

where $\rho_{\rm c} = 3H_0^2/(8\pi G) = 1.9 \times 10^{-29} h_0^2 \ {\rm g\ cm^{-3}}$ is the critical cosmological mass density. In Eq. (13) we have assumed a constant comoving electron density, i.e. $n_{\rm e}(z) \propto (1+z)^3$. Since some of the baryons are in bound systems this estimate should be taken as an upper limit.

Using the above we find that the mean plasma frequency in the intergalactic medium as a function of z may be approximated by

$$< \nu_{\rm p} >_{\rm IGM} (z) \approx 4.4 \ {\rm Hz} \left(\frac{x\Omega_{\rm b0} h_0^2}{0.02} \right)^{1/2} (1+z)^{3/2} \ . \quad (14)$$

The wavelength corresponding to 4.4 Hz is 6.8×10^4 km.

We can use these results to get a rough estimate of the dispersion time delay for broad band pulses. Following the same line of argument as before but with L of the order of the Hubble radius $R_{H_0} = c/H_0 \approx 3.0 h_0^{-1}$ Gpc, we find

$$\left(\frac{\Delta t_{\rm IGM}}{\rm min} \right) \approx 52 \ \left(\frac{\nu_1}{30 \ {\rm MHz}} \right)^{-2} \left[1 - \left(\frac{\nu_1}{\nu_2} \right)^2 \right] \left(\frac{h_0 L}{3 \ {\rm Gpc}} \right) \left(\frac{x\Omega_{\rm b0} h_0^2}{0.02} \right) \ , \quad (15)$$

where we have used Eqs. (8) and (14). At this stage we have not included any cosmological effects, which may change the result at most by a factor of a few. These will be discussed in §6. It must be emphasized that Eq. (15) probably represents an upper limit for $\Delta t_{\rm IGM}$.

Although we normalize the time delay to a frequency of 30 MHz in Eqs. (15) and (12), it should be pointed out that the Very Large Array (VLA) works at a frequency ten times higher. At 300 MHz the time delay is only 31 s. In addition, the observed radio afterglows of GRBs so far have fluxes of the order of μJy which are hard to detect with the VLA because of confusion. Measuring time delay of the radiation from extragalactic sources due to plasma dispersion may therefore be hard at present. However the time delay is definitely there, and should be observable in the future even if it requires new instruments, possibly in space.

4. Dispersion due to a finite photon mass

If the photon has a nonzero rest mass, m_γ, its total energy is $E = \gamma m_\gamma c^2$ where γ is the Lorentz factor corresponding to the photon's speed u_γ. Using $E = h\nu$, where ν is the frequency of the photon, and h is Planck's constant, we find that the speed can be written as (e.g. [11])

$$u_\gamma = c \left(1 - \left(\frac{\nu_\gamma}{\nu} \right)^2 \right)^{1/2} \ , \quad (16)$$

where

$$\nu_\gamma = \frac{m_\gamma c^2}{h} = 2.4 \ \mu{\rm Hz} \left(\frac{m_\gamma}{10^{-20} {\rm eV}/c^2} \right) \quad (17)$$

denotes a characteristic frequency associated with the photon's mass. The numbers in the second expression correspond to an upper limit of the photon mass of $10^{-20} \ {\rm eV/c^2} = 1.8 \times 10^{-53}$ g. The real observational upper limit may well be lower by 5 to 7 orders of magnitude [3], whereas the best upper limit from a time delay measurement is only 4.2×10^{-44} g from radio and gamma ray observations of GRB 980703 [26]. The wavelength corresponding to $2.4 \ \mu{\rm Hz}$ is 830 AU.

Equation (16) has the same form as the plasma relation (5) and for $\nu_2 > \nu_1 \gg \nu_\gamma$ we therefore find that the time delay is given by

$$\Delta t_\gamma \approx \frac{1}{2c} \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) \int_0^L \nu_\gamma^2 \, ds = \left(\frac{L}{2c} \right) \left(\frac{m_\gamma c^2}{h} \right)^2 \nu_1^{-2} \left[1 - \left(\frac{\nu_1}{\nu_2} \right)^2 \right] \quad (18)$$

or

$$\left(\frac{\Delta t_\gamma}{\rm ns} \right) \approx 1.0 \ \left(\frac{\nu_1}{30 \ {\rm MHz}} \right)^{-2} \left[1 - \left(\frac{\nu_1}{\nu_2} \right)^2 \right] \left(\frac{h_0 L}{3 \ {\rm Gpc}} \right) \left(\frac{m_\gamma}{10^{-20} {\rm eV}/c^2} \right)^2 \ . \quad (19)$$

Since cosmological corrections will increase this number at most by a factor of a few (see §6), it seems clear that other effects, in particular ordinary dispersion in the IGM will completely overwhelm dispersion due to a finite photon mass. We shall therefore not discuss this effect any further.

5. Dispersion due to quantum gravity effects

It has recently been suggested that quantum-gravitational effects may distort the standard dispersion relation for photons [1, 9]. Although this idea is highly controversial (see e.g. [25] and references therein) we shall consider it in this paper in the context of the dispersion time delay versus redshift test.

In scenarios of this type the speed of light in vacuum can approximately be written as

$$u_{\rm QG} = c\left(1 \pm \left(\frac{\nu}{\nu_{\rm QG}}\right)^{\eta}\right) \tag{20}$$

for energies much less than the Planck energy. Here we have introduced a characteristic frequency, $\nu_{\rm QG}$, associated with the quantum gravity energy scale, $E_{\rm QG}$, by

$$\nu_{\rm QG} = \chi\frac{E_{\rm QG}}{h} \approx 2.4\times 10^{33}\ {\rm GHz}\left(\frac{\chi E_{\rm QG}}{10^{19}\ {\rm GeV}}\right)\ . \tag{21}$$

In Eq. (20) η may be equal either to 1 or 2, the second value generally being considered more realistic. In most theories the energy scale is set by the Planck energy ($\approx 10^{19}$ GeV) but in some theories $E_{\rm QG}$ is much lower, even as low as $\sim 10^{16}$ GeV [29]. At present the strongest observational lower limit on E_{QG} is estimated to be $\sim 7\times 10^{15}$ GeV ([10] and references therein) The dimensionless factor χ in Eq. (21) has been introduced to take into account numerical factors that result from different theories. The wavelength corresponding to 2.4×10^{33} GHz is 1.3×10^{-34} m ($\approx$ the Planck length).

Comparing Eqs. (20) and (1) and setting $\nu_\star = \nu_{\rm QG}$ we see that

$$\alpha f(\nu,\nu_{\rm QG}) = \pm\left(\frac{\nu}{\nu_{\rm QG}}\right)^{\eta} \tag{22}$$

and for $\nu_{\rm QG} \gg \nu_2 > \nu_1$ we find from Eq. (4) that the time difference is given by

$$\Delta t_{{\rm QG}(\eta)} \approx \pm\left(\frac{1}{c}\right)\nu_2^{\eta}\left[1-\left(\frac{\nu_1}{\nu_2}\right)^{\eta}\right]\int_0^L \nu_{\rm QG}^{-\eta}\,ds = \pm\left(\frac{L}{c}\right)\left(\frac{h\nu_2}{\chi E_{\rm QG}}\right)^{\eta}\left[1-\left(\frac{\nu_1}{\nu_2}\right)^{\eta}\right]\ . \tag{23}$$

Hence if $\eta = 1$

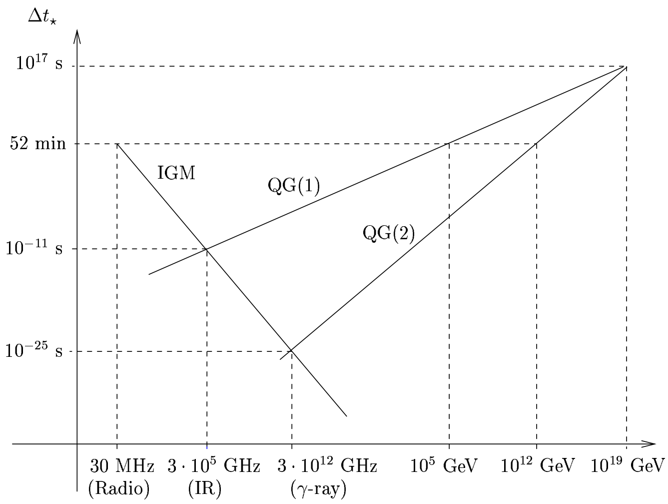
$$\left(\frac{\Delta t_{\rm QG(1)}}{\mu{\rm s}}\right) \approx \pm\,31\ \left(\frac{h_0 L}{3\ {\rm Gpc}}\right)\left(\frac{h\nu_2}{\rm MeV}\right)\left[1-\left(\frac{h\nu_1}{h\nu_2}\right)\right]\left(\frac{\chi E_{\rm QG}}{10^{19}\ {\rm GeV}}\right)^{-1}, \tag{24}$$

whereas if $\eta = 2$

$$\left(\frac{\Delta t_{\rm QG(2)}}{\rm s}\right) \approx \pm\,3\times 10^{-27}\left(\frac{h_0 L}{3\ {\rm Gpc}}\right)\left(\frac{h\nu_2}{\rm MeV}\right)^2\left[1-\left(\frac{h\nu_1}{h\nu_2}\right)^2\right]\left(\frac{\chi E_{\rm QG}}{10^{19}\ {\rm GeV}}\right)^{-2}, \tag{25}$$

where as before we have neglected cosmological effects, to be discussed in §6.

Comparing Eqs. (15), (24) and (25) and using $\chi E_{\rm QG} \sim 10^{19}$ GeV as well as $\nu_2 \gg \nu_1$ we find that the photon energy, $h\nu_2$, has to be of order 10^5 GeV for $\Delta t_{\rm QG(1)}$ to become as big as $\Delta t_{\rm IGM}$ is at the radio frequency $\nu_1 = 30$ MHz. In the case of QG(2) it has to be of order 10^{12} GeV. It is clear that ordinary IGM dispersion dominates QG(1) at radio wavelengths and well into the infrared, whereas in the case of QG(2) it dominates all the way to the gamma-ray part of the spectrum. This can also be seen in Fig. 1 which compares in a schematic way the relative importance of $\Delta t_{\rm IGM}$, $\Delta t_{\rm QG(1)}$ and $\Delta t_{\rm QG(2)}$ in the whole range from low frequency radio



Mynd 1. Schematic representation of the dispersion time delays $\Delta t_{\rm IGM} = \Delta t_{\rm IGM}(\nu_1)$ and $\Delta t_{{\rm QG}(\eta)} = \Delta t_{{\rm QG}(\eta)}(\nu_2)$, where we have assumed in both cases that $\nu_2 \gg \nu_1$ (see equations (15), (24), (25) and the text for further explanations). Parameters have their standard values: $\chi E_{\rm QG} = 10^{19}$ GeV, $h_0 L = 3$ Gpc and $x\Omega_{\rm b0}h_0^2 = 0.02$. Note that the extragalactic universe is opaque to gamma rays with $h\nu_2 > 10^5$ GeV.

waves to extremely high energy gamma rays. Note however that the extragalactic universe is opaque to gamma rays with energies of order 100 TeV or higher due to pair production when the gamma rays interact with background photons [13, 24]. Hence extremely high gamma ray energies ($h\nu_2 > 10^5$ GeV). can not be used for the dispersion time delays discussed in this paper.

There have already been several attempts to use observations of GRBs and other variable extragalactic sources to put limits on time delays that are not due to intrinsic variations of the sources or geometrical effects. These have in turn been used to set limits on the energy scale of quantum gravity, the photon mass and the fractional variation in the speed of light with frequency (see e.g. [10, 4, 17, 26, 9]). The high energies needed to test the dispersion due to quantum gravity effects (in the case of QG(1), since QG(2) effects will not be observable) will presumably be accessible with instruments aboard the GLAST space observatory which is scheduled for launch in September 2006.

6. Cosmological applications

In this section we will discuss how dispersion time delays of radiation from distant extragalactic sources can be used to differentiate between cosmological models. As far as we know this is the first systematic study of this new cosmological test, *dispersion time delay versus redshift*, although similar ideas have been discussed by Vertogradova et al. [28] in the limited case of dispersion due to quantum fluctuations in what they call the cosmic vacuum model.

We shall limit the discussion to expanding models of the Friedmann-Robertson-Walker type (see e.g. [24] for details). In these models cosmic evolution may be described by a dimensionless scale factor a which is a function of cosmic time t and takes the value 1 at the present epoch, t_0. It is useful for calculations to introduce a dimensionless time variable $\tau = H_0 t$ which measures cosmic time in units of the Hubble time, $1/H_0 = 9.8h_0^{-1}$ Gyr. It is relatively easy to see that for these models the Einstein field equations together with the equation of energy-momentum conservation can be reduced to the single equation (see e.g. [14]):

$$\frac{da}{d\tau} = \left\{ \Omega_{\rm m0}\left(\frac{1}{a}-1\right) + \Omega_{\rm r0}\left(\frac{1}{a^2}-1\right) + \Omega_{\Lambda 0}\left(a^2-1\right) + \Omega_{\rm Q0}\left(\frac{1}{a^{1+3w_{\rm Q}}}-1\right) + 1 \right\}^{1/2} , \qquad (26)$$

where $\Omega_{\rm m0}$, $\Omega_{\rm r0}$, $\Omega_{\Lambda 0}$ and $\Omega_{\rm Q0}$ are the present day values of the density parameters for matter (including dark matter), radiation, cosmological constant and quintessence respectively. The constant $w_{\rm Q}$ is the quintessence equation of state parameter defined by $P_{\rm Q} = w_{\rm Q}\rho_{\rm Q}c^2$, where $P_{\rm Q}$ and $\rho_{\rm Q}$ are the pressure and mass-energy density of the quintessence component respectively.

The redshift z of a given source is related to the value of a at the time of emission, $\tau_{\rm em}$, by

$$1 + z = \frac{1}{a(\tau_{\rm em})} \tag{27}$$

and pulses of radiation with observed frequencies ν_1 and ν_2 have frequencies $\nu_1/a(\tau)$ and $\nu_2/a(\tau)$ respectively at time $\tau_0 \geq \tau > \tau_{\rm em}$. Now suppose that the two pulses are emitted simultaneously by the source at time $\tau_{\rm em}$. If there is any dispersion we have from Eq. (1) that the difference in the speed of the pulses at time τ is

$$\Delta u = u_2 - u_1 \approx c\;\alpha[f(\nu_1/a, \nu_\star(a)) - f(\nu_2/a, \nu_\star(a))]\;, \tag{28}$$

where $a = a(\tau)$ and we have assumed that $|f(\nu/a, \nu_\star(a))| \ll 1$. The proper distance between the pulses then increases by an amount

$$dl = \Delta u\; dt\;, \tag{29}$$

in time dt. The corresponding dispersion time delay is

$$d(\Delta t_\star) \approx \frac{dl}{c} = \frac{\Delta u}{c} dt\;, \tag{30}$$

since both u_1 and u_2 are almost equal to c. Also $dt = (da/dt)^{-1}da = (H_0 da/d\tau)^{-1}da$ and therefore we have from Eq. (30) that

$$\Delta t_\star \approx \frac{R_{H_0}}{c}\int_{(1+z)^{-1}}^{1} \frac{\Delta u}{c}\left(\frac{da}{d\tau}\right)^{-1} da \approx \frac{R_{H_0}}{c}\int_{(1+z)^{-1}}^{1} \alpha[f(\nu_1/a, \nu_\star(a)) - f(\nu_2/a, \nu_\star(a))]\left(\frac{da}{d\tau}\right)^{-1} da\;. \tag{31}$$

This result will now be used to estimate the time delay in the expanding universe due to the main dispersion mechanisms, ordinary dispersion in the IGM at low energies and QG dispersion at high energies.

Bearing in mind that $\nu_{\rm IGM}(a) = \nu_{\rm IGM}(1)a^{-3/2}$, where $\nu_{\rm IGM}(1) = <\nu_{\rm p}>_{\rm IGM}(0)$ from Eq. (14), we find that

$$2c\;\Delta t_{\rm IGM}\;\left(\frac{\nu_1}{<\nu_{\rm p}>_{\rm IGM}(0)}\right)^2\left[1 - \left(\frac{\nu_1}{\nu_2}\right)^2\right]^{-1} \approx R_{H_0}\int_{(1+z)^{-1}}^{1} \frac{1}{a}\left(\frac{da}{d\tau}\right)^{-1} da\;. \tag{32}$$

Comparing this with our previous results we see that Eq. (15) can still be used for $\Delta t_{\rm IGM}$ if all the cosmological dependence is absorbed into L. We therefore replace L by an effective distance, $L_{\rm IGM}(z)$, given by

$$\frac{L_{\rm IGM}(z)}{R_{H_0}} = \left(\frac{h_0 L_{\rm IGM}(z)}{3.0\ \rm Gpc}\right) \approx \int_{(1+z)^{-1}}^{1} \frac{1}{a}\left(\frac{da}{d\tau}\right)^{-1} da\;. \tag{33}$$

It is interesting to note that $L_{\rm IGM}(z)$ so defined is equal to the proper distance to the source at the present epoch (see e.g. [14] for the relevant formulas).

The dispersion time delay versus redshift test consists in the application of Eqs. (32) and (33) to a set of sources with known redshifts and known arrival time differences between pulses at frequencies ν_1 and ν_2 $(> \nu_1)$. All the information about the cosmological model is contained in the integral on the right hand side of Eq. (33). As an example consider an Einstein-de Sitter universe. The integral is equal to $2[1 - (1+z)^{-1/2}]$ and has the value 0.85 if $z = 2$. This corresponds to $\Delta t_{\rm IGM} \approx 43$ minutes at $\nu_1 = 30$ MHz (with ν_2 assumed to be much higher).

Similarly, in the case of QG we find that

$$c\,\Delta t_{\mathrm{QG}(\eta)}\;\left(\frac{\nu_{\mathrm{QG}}}{\nu_2}\right)^{\eta}\left[1-\left(\frac{\nu_1}{\nu_2}\right)^{\eta}\right]^{-1}\approx R_{H_0}\int_{(1+z)^{-1}}^{1}\frac{1}{a^{\eta}}\left(\frac{da}{d\tau}\right)^{-1}da\;, \tag{34}$$

and hence $\Delta t_{\mathrm{QG}(\eta)}$ is still given by Eqs. (24) and (25) with L replaced by an effective distance, $L_{\mathrm{QG}(\eta)}(z)$, where

$$\frac{L_{\mathrm{QG}(\eta)}(z)}{R_{H_0}}=\left(\frac{h_0 L_{\mathrm{QG}(\eta)}(z)}{3.0\ \mathrm{Gpc}}\right)\approx\int_{(1+z)^{-1}}^{1}\frac{1}{a^{\eta}}\left(\frac{da}{d\tau}\right)^{-1}da\;. \tag{35}$$

If $\eta = 1$ we get the same result as in the IGM case (Eq. (33)), i.e. $L_{\mathrm{QG}(1)}(z) = L_{\mathrm{IGM}}(z) =$ present day proper distance to the source with redshift z. For an Einstein-de Sitter universe and $z = 2$ we therefore find that $\Delta t_{\mathrm{QG}(1)} \approx 26$ ms if $h\nu_2 = 1$ GeV, and $h\nu_1$ can be neglected. If $\eta = 2$ the integral equals $2[(1+z)^{1/2} - 1]$ for an Einstein-de Sitter universe and has value 1.46 at $z = 2$ which gives $\Delta t_{\mathrm{QG}(2)} \approx 4.4 \times 10^{-15}$ s for $h\nu_2 = 1$ TeV (assumed much greater than $h\nu_1$).

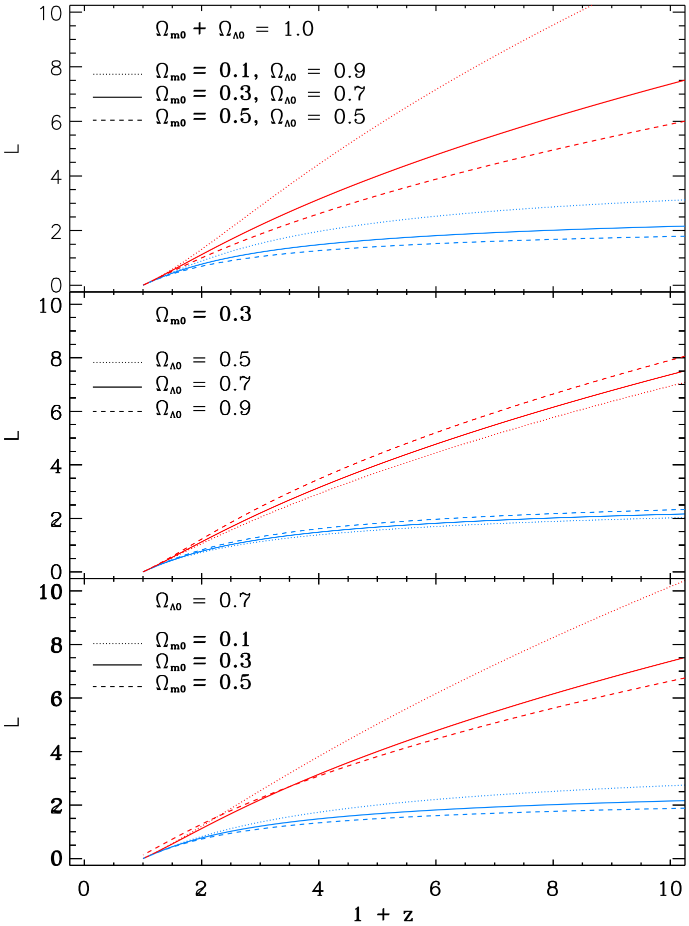
The top panel in Fig. 2 shows the effective distances $L_{\mathrm{IGM}}(z)$, $L_{\mathrm{QG}(1)}(z)$ and $L_{\mathrm{QG}(2)}(z)$ as functions of redshift for a flat universe. Note that $L_{\mathrm{IGM}} = L_{\mathrm{QG}(1)}$ and hence the behavior with redshift is identical (lower set of three curves). The effective distance $L_{\mathrm{QG}(2)}$ (upper set of three curves) behaves differently with redshift than the other two and shows more sensitivity to the values of $\Omega_{\mathrm{m}0}$ and $\Omega_{\Lambda 0}$. However, the time delay due to QG(2) dispersion is found by multiplying $L_{\mathrm{QG}(2)}$ by a very small number (see Eq. (25)), probably making the QG(2) test impossible in the near future. For practical applications one should therefore concentrate on the IGM and QG(1) tests (Eqs. (15) and (24)). The middle and bottom panels in Fig. 2 show the sensitivity of the effective distances to variations in $\Omega_{\mathrm{m}0}$ and $\Omega_{\Lambda 0}$ in more detail. One can clearly see that for realistic world models the tests are considerably more sensitive to $\Omega_{\mathrm{m}0}$ than to $\Omega_{\Lambda 0}$.

From Fig. 2 as well as the numerical examples above it seems clear that the dispersion time delay of explosive extragalactic events may not be easily observable. The factors multiplying the effective distances are small and in addition time delays due to other causes may dominate and they have to be corrected for in any case. Assuming that this can be done, one sees from Eq. (15) that in order to obtain an IGM dispersion time delay of the order of hours one needs to go to radio frequencies below 20 MHz. At such low frequencies reflection from the ionosphere makes measurements from the Earth's surface impossible. Similarly one can estimate that a QG(1) dispersion time delay of the order of hours requires photons with energies of 2 TeV or higher (see Eq. (24)), whereas for delays of the order of seconds the bounds on photon energies are much less severe, or around 30 GeV. Such energies and higher will be accessible with instruments aboard GLAST. We emphasize however that the extragalactic universe is opaque to gamma rays with energies in excess of 10^5 GeV because of pair production through interactions with background photons.

For the time delay versus redshift test to be a useful cosmological tool one must be certain that the radiation is either emitted simultaneously at the various frequencies involved or that the relative emission time can be estimated reliably. In the case of GRB afterglows e.g., one such signature might be the break in the light curve which in the fireball model is expected to occur simultaneously at all frequencies.

In the case of the IGM delay, one has in addition to correct for ISM dispersion, both in our own galaxy as well as in the host galaxies of the explosive events. To minimize this correction for our galaxy, sources at high galactic latitudes should preferably be chosen for the test. As far as the distant galaxies are concerned one must keep in mind that because of the cosmological redshift, a signal observed at Earth with frequency ν_1 propagated in the ISM of the distant galaxy with frequency $\nu_1(1+z)$ if the galaxy has redshift z. In addition one must take into account cosmological time dilation, $\Delta t'_{\mathrm{ISM}} = (1+z)\Delta t_{\mathrm{ISM}}$, where $\Delta t'_{\mathrm{ISM}}$ is the value observed on earth. When both effects are taken into account one finds that

$$\left(\frac{\Delta t'_{\mathrm{ISM}}}{\mathrm{min}}\right)\approx\frac{2.3}{(1+z)}\;\left(\frac{\nu_1}{30\ \mathrm{MHz}}\right)^{-2}\left[1-\left(\frac{\nu_1}{\nu_2}\right)^{2}\right]\left(\frac{L}{\mathrm{kpc}}\right)\left(\frac{< n_{\mathrm{e}} >_{\mathrm{ISM}}}{0.03\ \mathrm{cm}^{-3}}\right)\;, \tag{36}$$



Mynd 2. The effective distances $L_{\rm IGM} = L_{\rm QG(1)}$ (lower set of three curves) and $L_{\rm QG(2)}$ (upper set of three curves) in units of $R_{\rm H_0}$ as functions of $1 + z$ for various world models. The corresponding values of $\Omega_{\rm m0}$ and $\Omega_{\Lambda 0}$ are indicated in each panel. Other cosmological parameters ($\Omega_{\rm r0}$, $\Omega_{\rm Q0}$ and $w_{\rm Q}$) are all set equal to zero.

and hence the expansion of the universe helps to reduce this correction considerably, at least for high redshifts. Finally it should be emphasized that at high photon energies both ISM and IGM dispersion is negligible and hence no such correction is needed for X-rays or gamma rays.

7. Conclusions

In this paper we have compared various dispersion mechanisms and estimated the dispersion time delay of electromagnetic pulses from extragalactic sources, such as GRBs and their afterglows as well as flares in active galactic nuclei. We find that time delays due to ordinary dispersion in the IGM could be of order of hours for pulses at low radio frequencies as compared with pulses of optical light or higher energy photons from the same events. However, at these frequencies ionospheric reflection severely restricts measurements from the Earth's surface. If real, the recently proposed QG(1) dispersion time delay at very high energies is more promising and it might be measurable with GLAST under favorable circumstances.

We have also proposed a new cosmological test using dispersion time delay as a function of redshift to differentiate between world models. Provided that some reliable method can be found to estimate the relative emission time of pulses at different frequencies, the test could be profitably used by comparing the arrival time of high and low energy pulses, e.g. from explosive events in cosmologically distant sources.

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